Analytic Geometry Review

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad r^2 = x^2 + y^2$ $\quad y = mx + b$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad MP\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

1. Given the line 3x + 5y – 10 = 0 determine the slope.

$3x + 5y - 10 = 0$

$\frac{5y}{5} = \frac{-3x}{5} + \frac{10}{5}$

$y = -\frac{3}{5}x + 2$

∴ the slope of the line is $-\frac{3}{5}$

2. What is the name of the triangle where all sides have the same length?

∴ the triangle is an equilateral

3. What is the distance between points A(3,-5) and B(-6,7)?

$d = \sqrt{(7-(-5))^2 + (-6-(3))^2}$

$= \sqrt{(12)^2 + (9)^2}$

$= \sqrt{144+81}$

$= \sqrt{225}$

$= 15$

∴ the distance between A and B is 15 units

4. Determine the slope of a line perpendicular to the line $\frac{2}{3}x - y = 8$

$\frac{2}{3}x - y = 8$

$\frac{2}{3}x - 8 = y$

$y = \frac{2}{3}x - 8$

$m_\perp = -\frac{3}{2}$

∴ the perpendicular line has a slope of $-\frac{3}{2}$

5. Determine the length of the line segment from C(-13,7) to D(5,20)

$d = \sqrt{(5-(-13))^2 + (20-(7))^2}$

$= \sqrt{(18)^2 + (13)^2}$

$= \sqrt{324+169}$

$= \sqrt{493}$ → $d = 22.20$

∴ the length of the line segment is 22.20 units

Determine the midpoint of the line segment joining C(2,1) and D(-2,9)

$MP_{CD}\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{2+(-2)}{2}, \frac{1+9}{2}\right)$

$\left(\frac{0}{2}, \frac{10}{2}\right)$

$(0,5)$

∴ the midpoint of CD is (0,5)

6. ΔKLM has vertices K(-1,2), L(2,3), and M(-3,4). Find the equation for the median from K to ML.

① $MP_{ML}\left(\frac{2+(-3)}{2}, \frac{3+4}{2}\right)$

$\left(-\frac{1}{2}, \frac{7}{2}\right)$

$(-0.5, 3.5)$

② $m_{K(MP)} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{3.5-(2)}{-0.5-(-1)}$

$= \frac{1.5}{0.5}$

$= 3$

③ $y = 3x + b$

④ sub K(-1,2) into $y=3x+b$

$y = 3x+b$

$2 = 3(-1)+b$

$2+3 = b$

$b = 5$

∴ the equation of the median is $y=3x+5$

7. The vertices of the ΔDEF are D(4,2), E(-2,-2), and F(2,-8). Classify the triangle by side length

$FD = \sqrt{(4-(2))^2+(2-(-8))^2}$

$= \sqrt{(2)^2+(10)^2}$

$= \sqrt{4+100}$

$= \sqrt{104}$

$\doteq 10.19$

$DE = \sqrt{(4-(-2))^2+(2-(-2))^2}$

$= \sqrt{(6)^2+(4)^2}$

$= \sqrt{36+16}$

$= \sqrt{52}$

$\doteq 7.211$

$FE = \sqrt{(-2-(2))^2+(-2-(-8))^2}$

$= \sqrt{(-4)^2+(6)^2}$

$= \sqrt{16+36}$

$= \sqrt{52}$

$\doteq 7.211$

∴ it is an isosceles triangle because side DE = FE.

8. Show that the quadrilateral CDEF with vertices C(-11,-1), D(9,4), E(1,8), and F(-19,3) is a parallelogram.

$m_{DE} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{8-(4)}{1-(9)}$

$= \frac{4}{-8}$

$= -\frac{1}{2}$

$m_{CF} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-1-(3)}{-11-(-19)}$

$= \frac{-4}{8}$

$= -\frac{1}{2}$

$m_{CD} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{4-(-1)}{9-(-11)}$

$= \frac{5}{20}$

$= \frac{1}{4}$

$m_{FE} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{8-(3)}{1-(-19)}$

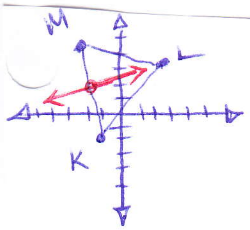
$= \frac{5}{20}$

$= \frac{1}{4}$

∴ it is a parallelogram because opposite sides are parallel but not perpendicular

+ distance OR diagonals ⊥

9. ΔKLM has vertices K(-1,-2), L(2,3), and M(-3,4). Find an equation for the right bisector of MK.



① MP_{MK} $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{-3+(-1)}{2}, \frac{4+(-2)}{2}\right)$

$\left(\frac{-4}{2}, \frac{2}{2}\right)$

$(-2, 1)$

② $m_{MK} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{4-(-2)}{-3-(-1)}$

$= \frac{6}{-2}$

$= -3$

$m_\perp = \frac{1}{3}$

③ $y = \frac{1}{3}x + b$ sub in MP

$1 = \frac{1}{3}(-2)+b$

$1 = -\frac{2}{3}+b$

$b = +\frac{2}{3}+1$

$b = \frac{2}{3}+\frac{3}{3}$

$b = \frac{5}{3}$

∴ the equation of the right bisector is $y = \frac{1}{3}x + \frac{5}{3}$

10. Write an equation for the circle with centre (0, 0) and given radius.

a) Radius 13
b) Radius 4.5

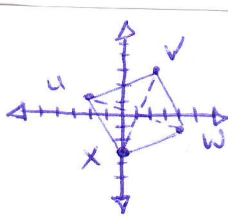
a) $x^2+y^2=r^2$
$x^2+y^2=13^2$
$x^2+y^2=169$

b) $x^2+y^2=r^2$
$x^2+y^2=4.5^2$
$x^2+y^2=20.25$

11. Determine the radius of the circle with centre (0, 0) and given equation $x^2 + y^2 = 1.69$

$r^2 = x^2+y^2$
$r^2 = 1.69$
$\sqrt{r^2} = \sqrt{1.69}$
$r = \pm 1.3$
∴ the radius is $+1.3$.

12. A square has vertices at U(-2,1), V(2,3), W(4,-1) and X(0,-3). Verify that the diagonals perpendicularly bisect each other.



find VX & UW
find POI.

∴ the diagonals perpendicularly bisect each other since $-\frac{1}{3}$ is the negative reciprocal of 3.

① $m_{VX} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-3-(3)}{0-2}$

$= \frac{-6}{-2}$

$= 3$

$m_{UW} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-1-(1)}{4-(-2)}$

$= \frac{-2}{6}$

$= -\frac{1}{3}$

13. Verify that the quadrilateral with vertices O(0,0), P(3,5), Q(13,7, and R(5,1) is a trapezoid

$m_{OP} = \frac{y_2-y_1}{x_2-x_1}$
$= \frac{5-(0)}{3-(0)}$
$= \frac{5}{3}$

$m_{PQ} = \frac{y_2-y_1}{x_2-x_1}$
$= \frac{7-(5)}{13-(3)}$
$= \frac{2}{10}$
$= \frac{1}{5}$

$m_{QR} = \frac{y_2-y_1}{x_2-x_1}$
$= \frac{7-(1)}{13-(5)}$
$= \frac{6}{8}$
$= \frac{3}{4}$

$m_{RO} = \frac{y_2-y_1}{x_2-x_1}$
$= \frac{1-0}{5-0}$
$= \frac{1}{5}$

Since PQ and OR have slopes of 1/5 and so parallel; the other two sides have different slopes.

$\therefore$ OPRQ is a trapezoid

14. Find the shortest distance from the given point to the given line. Round to the nearest tenth, if necessary. (1,4) and y = x -5

① $m_{\perp} = -1$

② $y = -x + b$

③ sub (1,4) in $y = -x + b$
$y = -x + b$
$4 = -(1) + b$
$b = 4 + 1$
$b = 5$

① $y = -x + 5$
② $+ \;\; y = x - 5$
$\frac{2y}{2} = \frac{0}{2}$
$y = 0$

$\therefore$ the shortest distance is 5.7 units

sub y = 0 into ①
$0 = -x + 5$
$x = 5$
$\therefore$ the POI is (5,0)

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(5-1)^2 + (0-4)^2}$
$= \sqrt{4^2 + (-4)^2}$
$= \sqrt{16+16}$
$= \sqrt{32}$
$\rightarrow \doteq 5.7$

15. ΔKLM has vertices K (-1, -2), L (2, 3), and M (-3, 4). Find an equation for MP, the altitude from M to KL

$$M_{KL} = \frac{y_2-y_1}{x_2-x_1} = \frac{3-(-2)}{2-(-1)} = \frac{5}{3}$$

$$m_{\perp} = -\frac{3}{5}$$

$$y = -\frac{3}{5}x + b$$

sub in M

$$y = -\frac{3}{5}x + b$$
$$4 = -\frac{3}{5}(-3) + b$$
$$4 = +\frac{9}{5} + b$$
$$b = 4 - \frac{9}{5}$$
$$b = \frac{20}{5} - \frac{9}{5}$$
$$b = \frac{11}{5}$$

∴ equation of the altitude is

$$y = -\frac{3}{5}x + \frac{11}{5}$$

16. A university has three student residences, which are located at points A(2,2), B(10,6), and C(4,8) on a grid. The university wants to build a tennis court an equal distance from all three residences. Determine the coordinates of the tennis courts.

$$M_{AB} = \left(\frac{10+2}{2}, \frac{6+2}{2}\right) = (6, 4)$$

$$\text{Slope}_{AB} = \frac{6-2}{10-2} = \frac{4}{8} = \frac{1}{2}$$

$$m_{\perp} = -2$$

$y = -2x + b$ sub (6,4)

$$4 = -2(6) + b$$
$$4 + 12 = b$$
$$b = 16$$

① $\boxed{y = -2x + 16}$

$$M_{BC} = \left(\frac{10+4}{2}, \frac{6+8}{2}\right) = \left(\frac{14}{2}, \frac{14}{2}\right) = (7, 7)$$

$$\text{slope}_{BC} = \frac{8-6}{4-10} = \frac{2}{-6} = -\frac{1}{3}$$

$$m_{\perp} = +3$$

$y = 3x + b$ sub (7,7)

$$7 = 3(7) + b$$
$$7 = 21 + b$$
$$7 - 21 = b$$
$$b = -14$$

② $\boxed{y = 3x - 14}$

$$3x - 14 = -2x + 16$$
$$3x + 2x = 16 + 14$$
$$\frac{5x}{5} = \frac{30}{5}$$
$$x = 6$$

sub x = 6 into ②

$$y = 3x - 14 = 3(6) - 14 = 18 - 14 = 4$$

∴ the tennis court should be built at (6, 4)

17. ▲LMN has vertices ae L(-5,4), M(2,-3), and N(1,4). Use analytic geometry to determine:

a. the coordinates of the centroid

$\left\{ \frac{-5+2+1}{3}, \frac{4+(-3)+4}{3} \right\}$

$\left\{ -\frac{2}{3}, \frac{5}{3} \right\}$

b. the coordinates of the orthocentre

$m_{LM} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-3-4}{2-(-5}$

$= \frac{-7}{7}$

$m_\perp = 1$

sub (1,4) to solve b

$y = x+b$

$4 = 1+b$

$b = 4-1$

$b = 3$

① $\boxed{y = x+3}$

$m_{MN} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{4-(-3)}{1-2}$

$= \frac{7}{-1}$

$= -7$

$m_\perp = \frac{1}{7}$

sub (-5,4) to solve b

$y = \frac{1}{7}x + b$

$4 = \frac{1}{7}(-5)+b$

$4 = -\frac{5}{7}+b$

$b = 4+\frac{5}{7}$

$b = \frac{4\times7}{1\times7}+\frac{5}{7}$

$= \frac{28}{7}+\frac{5}{7}$

$= \frac{33}{7}$

② $\boxed{y = \frac{1}{7}x + \frac{33}{7}}$

Solve POI by elimination

$\left[y = \frac{1}{7}x + \frac{33}{7}\right] \times 7$

② $7y = x+33$

$-$ ① $y = x+3$

$6y = 30$

$\frac{6y}{6} = \frac{30}{6}$

$y = 5$

sub y = 5 into ①

$5 = x+3$

$-3+5 = x$

$x = 2$

∴ the orthocentre is (2,5)

18. ▲ABC has vertices ae A(4,2), B(0,4), and C(2,-2). Use analytic geometry to determine:

a. the coordinates of the circumcentre

$Mp_{AB} = \left\{ \frac{4+0}{2}, \frac{2+4}{2} \right\}$

$= (2,3)$

$\text{Slope}_{AB} = \frac{4-(2)}{0-(4)}$

$= -\frac{1}{2}$

$m_\perp = 2$

sub (2,3) to solve b

$y = 2x+b$

$3 = 4+b$

$b = 3-4$

$b = -1$

$\boxed{① \; y = 2x-1}$

$Mp_{BC} = \left\{ \frac{0+2}{2}, \frac{4+(-2)}{2} \right\}$

$= (1,1)$

$\text{Slope}_{BC} = \frac{4-(-2)}{0-(+2)}$

$= -3$

$m_\perp = \frac{1}{3}$

sub (1,1) to solve b

$y = \frac{1}{3}x + b$

$1 = \frac{1}{3}(1)+b$

$b = 1-\frac{1}{3}$

$b = \frac{3}{3}-\frac{1}{3}$

$b = \frac{2}{3}$

$\boxed{② \; y = \frac{1}{3}x + \frac{2}{3}}$

Solve POI by elimination

①$[y = 2x-1] \times 3$

②$\left[y = \frac{1}{3}x + \frac{2}{3}\right] \times 3$

① $3y = 6x-3$

$-$ ② $3y = x+2$

$0y = 5x-5$

$\frac{-5x}{-5} = \frac{-5}{-5}$

$x = 1$

sub x = 1 into ①

$y = 2x-1$

$y = 2(1)-1$

$y = 1$

∴ the coordinate of the circumcentre is (1,1)